Determination of field intensities belonging to the wedge regions adjacent to a convex triangular obstacle subject to axially independent conditions

Sanjay Kumar

P. G. Centre Department of Mathematics, Ram Dayalu Singh College,
Muzaffarpur-842002, Bihar, India
e-mail : dr.sanjaykumar06@gmail.com.

Abstract : The present paper gives an interaction of standing electromagnetic waves with a smooth convex triangular obstacle K and its adjacent wedge regions. The concerning electromagnetic fields are supposed to be independent of the variations along the axis of K. Governing Helmholtz wave equation, being resulted from the Maxwell's field equations, have been encountered subject to initial boundary conditions of the field intensities on the wedge surfaces ∂K . The concerning boundary value problem has been particularly associated with Dirichlet and Neumann conditions on ∂K , giving rise to Dual-Bessel series relations. The unknown coefficients of the said dual series relations have been determined by making use of Lommel's integral for a pair of Bessel functions of the first kind. Two existence theorems regarding the cylindrical mode of polarisation of electromagnetic wave have established, furnishing thereby the components electric and magnetic field intensity vectors. Wave characteristics like reflection and transmission have been determined on the basis of said existence theorem. Finally the expressions of the field intensities $\boldsymbol{H}$ and $\boldsymbol{E}$ have been utilized for determining the current density.

Keywords : Electromagnetic field intensities, convex triangular prism, Maxwell's equations.

1. Introduction

A convex triangular obstacle forms a vital part of a periodic echellete grating. In recent years [1-7] quite a good number of results have been reported pertaining to the groove field estimates and the efficiency of the said grating. The present paper deals with a general convex triangular prismatic obstacle K having an open base, a flare angle β, the groove depth 'h' and the grating period 'd' Figure 1. The bounding faces ∂K of the obstacle K and its adjacent wedge surfaces Figure 2 are subjected to reflection, transmission and grazing due to an axially independent EM wave. EM field intensity $\boldsymbol{F} = \left(\boldsymbol{H} \vee \boldsymbol{E}\right)$ are derived from the governing Maxwell's equations

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} = \sigma \boldsymbol{E} + \in \frac{\partial \boldsymbol{E}}{\partial t},$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} = -\mu \frac{\partial \boldsymbol{H}}{\partial t}$$

and

$$\nabla^2 \boldsymbol{F} = \mu\left(\sigma \frac{\partial \boldsymbol{F}}{\partial t} + \in \frac{\partial^2 \boldsymbol{F}}{\partial t^2}\right)$$

where $\boldsymbol{H}$ and $\boldsymbol{E}$ stand for the magnetic and the electric intensity vectors. The physical elements $\sigma, \in,$ μ, $\boldsymbol{J}$ and $\boldsymbol{B}$ stand for conductivity, permittivity, permeability, current density and magnetic flux density associated with 'M', respectively. Maxwell's equations have been encountered subject to prescribed initial boundary conditions of the EM field on ∂K. The concerning boundary value problems happens to be associated with the Dirichlet's conditions and the Neumann's conditions initially (t = 0) on ∂K. Dirichlet's problem is an example of well posed boundary value problems as observed earlier [8-10]. An axially independent field intensity satisfies the condition $\partial \boldsymbol{F} / \partial x_3 = 0$ which leads to the independence of $\boldsymbol{F}$ relative to the directions parallel to the edges OO', AA' and BB' of the model 'M'. As such, a cylindrical wave function happen to exist as a solution of the Maxwell's equation subject to cylindrical coordinate transformation $x_1 = \rho \cos\phi, x_2 = \rho \sin\phi, x_3 = x_3$. In particular a cylindrical wave is said to be axially independent whenever the associated wave function is independent of the z coordinate. Hence an axially independent cylindrical wave has been arrived in the form of the Fourier-Bessel series [11-12]

$$\boldsymbol{F} = \boldsymbol{F}(\rho,\phi) = \sum_{i \in J^{+}} \boldsymbol{A}_i J_{\eta}(\rho k_i) \exp\{(j\omega - (\sigma/2 \in))t + j\eta\phi\}$$

where $\eta \geq 1$ and k_i is the ith wave number in a certain frequency range associated with interacting EM waves. The unknown coefficients $\boldsymbol{A}_i$ happen to satisfy two pairs of dual-Bessel series relations in the wedge regions R_i (i =1, 2). Oblique coordinate transformation [13] being associated with the geometry of M have been found to be of great value for evaluating the coefficients $\boldsymbol{A}_i$. Finally, the expressions of $\boldsymbol{F}$ have been used for computing the current density.

2. Formulation of the problem

Consider the Maxwell's equation

$$\nabla^2 \boldsymbol{F} = \frac{\partial^2 \boldsymbol{F}}{\partial x_1^2} + \frac{\partial^2 \boldsymbol{F}}{\partial x_2^2} + \frac{\partial^2 \boldsymbol{F}}{\partial x_3^2} = \mu\left(\sigma \frac{\partial \boldsymbol{F}}{\partial t} + \in \frac{\partial^2 \boldsymbol{F}}{\partial t^2}\right) \qquad (1)$$

where $\boldsymbol{F} = (\boldsymbol{H} \vee \boldsymbol{E}) = \boldsymbol{F}(x_1, x_2, x_3, t)$ stands for vector field intensity. Transforming (1) by using cylindrical coordinates $x_1 = \rho\cos\phi,\ x_2 = \rho\sin\phi,\quad x_3 = z$ subject to the axially independent condition $\frac{\partial \boldsymbol{F}}{\partial x_3} = 0$, one can arrive at the equation

$$\nabla^2 \boldsymbol{F} = \left[\frac{\partial^2 \boldsymbol{F}}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial \boldsymbol{F}}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 \boldsymbol{F}}{\partial \phi^2}\right] = \mu\left(\sigma \frac{\partial \boldsymbol{F}}{\partial t} + \in \frac{\partial^2 \boldsymbol{F}}{\partial t^2}\right) \qquad (2)$$

Now, applying variable separable method for the equation (2), one can arrive at the solution

$$F = F_1(\rho)F_2(\phi)F_3(t)$$

where F_1, F_2 and F_3 satisfy the ordinary differential equations

$$\rho^2 F_1'' + \rho F_1' + (k^2\rho^2 - \eta^2)F_1 = 0 \qquad (3)$$

$$F_2'' + \eta^2 F_2 = 0 \qquad (4)$$

and

$$\mu\left(\in F_3'(t) + \sigma F_3'(t)\right) + k^2 F_3 = 0 \qquad (5)$$

The equations (3), (4) and (5) furnish the solutions

$$F_1(\rho) = J_\eta(k\rho),\ F_2(\phi) = Ae^{\eta\phi J} \text{ and } F_3 = Be^{-\sigma t/2\in} \tag{6}$$

$$\mu^2\sigma^2 - 4\mu \in k^2 = -4\omega^2\mu^2c^2 \tag{6a}$$

where $J_\eta(\rho k)$ is the Bessel function of the first kind of order η, and A and B are arbitrary constants. The solution (6) of the wave equation (2) would give rise to an axially independent cylindrical wavelet

$$\boldsymbol{F} = J_\eta(k\rho)\exp\{(j\omega - (\sigma/2\in))t + j\eta\phi\} \tag{7}$$

associate with the frequency ω and the wave number k, satisfying the non linear relation (6).

Fourier-Bessel series for the solution (7) :

In order to match the solution (7) on the boundaries K of the model 'M' it is essential to sum up the same solution in the form of Fourier-Bessel series

$$\boldsymbol{F} = \sum_{i\in J^+} \boldsymbol{A}_i J_\eta(k_i\rho)\exp\{j\eta\phi + (j\omega - (\sigma/2\in))t\} \tag{8}$$

Now, assuming Dirichlet's conditions

$$\left.\begin{array}{ll} F\big|_{OA} = F_1(x',0,0), & F\big|_{AC} = F_2(a,y',0) \\ F\big|_{OB} = F_3(0,y',0) \text{ and } & F\big|_{BC'} = F_4(x',-b,0) \end{array}\right\} \tag{9}$$

on the faces OA, AC, OB and BC` of the model M, one can arrive at the following pair of dual series relation by matching the Fourier-Bessel series (8) with the function $F_i(i = 1,2,3,4)$ given by (9) for $t = 0$

$$\begin{aligned} &\sum_{i\in J^+} \boldsymbol{A}_i J_\eta(k_i\rho) = F_1(x',0,0)e^{-\eta\phi J} \quad \text{for } 0 \le \rho \le a \\ &\sum_{i\in J^+} \boldsymbol{A}_i J_\eta(k_i\rho) = F_2(a,y',0)e^{-\eta\phi J} \quad \text{for } a \le \rho \le d \end{aligned} \tag{10}$$

Now, using the oblique transformation [13]

$$\begin{aligned} x_1 &= \rho\cos\phi = x'\cos\theta_0 - y'\cos(\theta_0 + \beta) \\ x_2 &= \rho\sin\phi = -x'\sin\theta_0 + y'\cos(\theta_0 + \beta) \end{aligned} \tag{11}$$

One can arrive at the coordinates

$$\begin{aligned}
x' &= \rho \sin(\theta_0 + \phi + \beta)/\sin\beta \\
y' &= \rho \sin(\theta_0 + \phi)/\sin\beta \\
\rho_{AC} &= \rho\big|_{FaceAC} = a \sin\beta/\sin(\theta_0 + \phi + \beta) \\
\rho_{BC'} &= \rho\big|_{FaceBC'} = -b \sin\beta/\sin(\theta_0 + \phi)
\end{aligned}$$

Hence one, can further express the dual equations in the form

$$\sum_{i \in J^+} \boldsymbol{A}_i J_\eta(k_i\rho) = f(\rho) \text{ for } 0 \le \rho \le d \tag{12}$$

where

$$f(\rho) = e^{\eta\theta j} F_1(\rho,0,0) \text{ for } 0 \le \rho \le a$$

and

$$e^{-\eta\phi J} F_2(a, y'_{AC}, 0) \text{ for } a \le \rho \le d \quad (-\theta \le \phi \le 0)$$

Now using Lommel's integral [14] for orthogonality of $J_\eta(k_i\rho)$ in the interval $0 \le \rho \le d$ one can express A_i in the form

$$\frac{1}{2}\boldsymbol{A}_i d^2 J_{\eta+1}^2(dk_i) = \int_{\rho=0}^{d} \rho\, J_\eta(k_i\rho) f(\rho) d\rho \tag{13}$$

Where k_i is the i^th^ positive zero of $J_n(dx) = 0$, Combining (12) and (13) one can further arrive at the result

$$\begin{aligned}
\frac{1}{2}\boldsymbol{A}_i d^2 J_{\eta+1}^2(dk_i) = e^{\eta\theta j} &\int_{\rho=0}^{a} \rho\, J_\eta(k_i\rho) F_1(\rho,0,0) d\rho \\
&+ \int_{\rho_{AC}=a}^{d} \rho_{AC}\, e^{-\eta\phi J} J_\eta(k_i\rho_{AC}) F_2(a, y'_{AC}, 0) d\rho_{AC}
\end{aligned} \tag{14}$$

Now, combining (13) and (14) the unknown coefficients '$\boldsymbol{A}_i$' may be precisely determined by means of the formula

$$\frac{1}{2}\boldsymbol{A}_i d^2 J_{\eta+1}^2(dk_i) A_i = e^{\eta\theta j} \int_{\rho=0}^{a} \rho\, J_\eta(k_i\rho) F_1(\rho,0,0) d\rho + I_1$$

where

$$I_1 = e^{\eta J(\theta+\beta)} (a \sin\beta)^2 \int_{t=\cos ec\beta}^{\cos ec(\theta+\beta)} t \exp\left(-\eta J \cos es^{-1} t\right) J_\eta(k_i a \sin\beta t) F_2(a, y'_{AC}, 0) dt \tag{15}$$

Neumann's conditions on the boundaries ∂K :

Assuming Neumann's conditions

$$\begin{aligned} \left.\frac{\partial F}{\partial n_1}\right|_{OA} &= G_1(x',0,0), \left.\frac{\partial F}{\partial n_2}\right|_{AC} = G_2(a,y',0) \\ \left.\frac{\partial F}{\partial n_2}\right|_{OB} &= G_3(0,y',0), \left.\frac{\partial F}{\partial n_1}\right|_{BC'} = G_4(x',-b,0) \end{aligned} \tag{16}$$

on the faces OA, AC, OB and BC` of the model 'M' one can arrive at the following pairs of dual equations

$$\begin{aligned} &\sum_{i\in J^+} \boldsymbol{A}_i \frac{\partial}{\partial n_1}\left\{e^{-\eta\theta j} J_\eta(\rho k_i)\right\} = G_1(\rho,0,0)\Big|_{0\le\rho\le d} \\ &\sum_{i\in J^+} \boldsymbol{A}_i \frac{\partial}{\partial n_2}\left\{e^{-\eta\theta J} J_\eta(\rho_{AC} k_i)\right\} = G_2(a,\rho_{AC},0)\Big|_{a\le\rho_{AC}\le d} \end{aligned} \tag{17}$$

and

$$\begin{aligned} &\sum_{i\in J^+} \boldsymbol{A}_i \frac{\partial}{\partial n_2}\left\{e^{-\eta(\theta+\beta)j} J_\eta(\rho k_i)\right\} = G_3(0,\rho,0)\Big|_{0\le\rho\le b} \\ &\sum_{i\in J^+} \boldsymbol{A}_i \frac{\partial}{\partial n_1}\left\{e^{-\eta(\theta+\phi+\beta)j} J_\eta(\rho_{BC'} k_i)\right\} = G_4(\rho_{BC'}-b,0)\Big|_{b\le\rho_{BC'}\le d} \end{aligned} \tag{18}$$

where the normal derivatives $\frac{\partial}{\partial n_i}(i=1,2)$ may be expressed in the equivalent form

$$\frac{\partial}{\partial n_1} = \sin(\theta+\phi)\frac{\partial}{\partial\rho} + \rho^{-1}\cos(\theta+\phi)\frac{\partial}{\partial\phi}$$

and $$\frac{\partial}{\partial n_2} = -\sin(\theta+\phi+\beta)\frac{\partial}{\partial\rho} + \rho^{-1}\cos(\theta+\phi+\beta)\frac{\partial}{\partial\phi} \tag{19}$$

by means of the oblique transformations (11).

Hence, combining (17) and (18) with (19) successively, one can arrive at the following pair of dual series relations :

$$\begin{aligned} &\sum_{i\in J^+} \boldsymbol{A}_i J_\eta(k_i\rho) = G'(\rho)\Big|_{0\le\rho\le a} \\ &\sum_{i\in J^+} \boldsymbol{A}_i F_\eta^{H_1}(\rho_{AC}k_i) = G^2(\rho_{AC},\ \phi)\Big|_{0\le\rho_{AC}\le d} \end{aligned} \tag{20}$$

and

$$\begin{aligned}&\sum_{i\in J^+}\boldsymbol{A}_i J_\eta(k_i\rho)=G^3(\rho)\Big|_{0\le\rho\le b}\\&\sum_{i\in J^+}\boldsymbol{A}_i F_\eta^{H_2}(\rho_{BC'},k_i)=G^4(\rho_{BC'},d)\Big|_{b\le\rho_{BC'}\le d}\end{aligned}\tag{21}$$

where $\rho_{AC}=a\sin\beta/\sin(\theta+\beta+\phi),\ \rho_{BC'}=b\sin\beta/\sin(\alpha+\beta)$

$-a\le\phi\le 0,\ 0\le\alpha\le\pi-\theta-\beta,\ a\le\rho_{AC}\le d, b\le\rho_{BC'}\le d, H_1=\eta\dot{J}\cot(\theta+\beta+\phi)$

Making use of the recurrence relation [7]

$$\eta J_\eta(\rho k_i)+\rho k_i J'_\eta(\rho k_i)=\rho k_i J_{\eta-1}(\rho k_i)$$

the second equation in (20) can be expressed in the form

$$\begin{aligned}\sum_{i\in J^+}\boldsymbol{A}_i F_\eta^{H_1}(\rho_{AC}k_i)&=\sum_{i\in J^+}\boldsymbol{A}_i\left[(H_1-\eta)J_\eta(\rho_{AC}k_i)+\rho_{AC}k_i J_{\eta-1}(\rho_{AC}k_i)\right]\\&=G^2(\rho_{AC},\phi)\end{aligned}\tag{22}$$

Now, imagine an unknown step function $h_1(\rho)$ satisfying the equation

$$\sum_{i\in J^+}\boldsymbol{A}_i k_i J_{\eta-1}(\rho_{AC}k_i)=h_1(\rho)\quad(0\le\rho\le d)\tag{23}$$

such that

$$h_1(\rho)=f_1(\rho)\text{ for }0\le\rho\le a\text{ and }g_1(\rho_{AC},\phi,0)\text{for }a\le\rho_{AC}\le d\tag{24}$$

and $g_1(\rho_{AC},\phi,0)$ is supposed to be continuous at the corner ‘A’ of the wedge region R_1 : Δ OAC.

Again, using the theory of Fourier-Bessel series, and can evaluate ‘A_i’ by means of the formula

$$\boldsymbol{A}_i=\frac{2}{d^2k_iJ_\eta^2(dk_i)}\left[\int_{\rho_{AC}=a}^{d}\rho_{AC}J_{\eta-1}(\rho_{AC}k_i)g_1(\rho_{AC},\phi,0)d\rho_{AC}+\int_{\rho=0}^{a}J_{\eta-1}(\rho k_i)f_1(\rho)d\rho\right]$$

(25)

where k_i is the i^th^ positive root of the transcendental equation $J_{\eta-1}(dk)=0$.

Axially independent cylindrical wave functions and the components of electric and magnetic intensities vectors :

The expression (8) represents a cylindrical wave function

$$\Phi(\rho,\phi,t)=\Phi^F(\rho,\phi)e^{-\sigma t/2\in}e^{J\omega t}\tag{26}$$

where $\Phi^F(\rho,\phi)=\sum_{i\in J^+}\boldsymbol{A}_i(F)J_\eta(\rho k_i)e^{\eta\phi J}$ stands for the free space axially independent cylindrical wave associated with the frequency 'ω' and the wave number *k*, satisfying the non-linear relation (6a).

Now, recalling the Maxwell's equations $\nabla\times\boldsymbol{H}=\sigma\boldsymbol{E}+\in\frac{\partial\boldsymbol{E}}{\partial t}$ and $\nabla\times\boldsymbol{E}=-\mu\frac{\partial\boldsymbol{H}}{\partial t}$ subject to the axially independent condition $\frac{\partial\boldsymbol{F}}{\partial x_3}=0$ one can arrive at the following relations :

$$\begin{aligned}E_1\{\sigma+\in(\omega\dot{J}-\sigma/2\in)\}&=\frac{\partial H_3}{\partial x_2}\\E_2\{\sigma+\in(\omega\dot{J}-\sigma/2\in)\}&=\frac{\partial H_3}{\partial x_1}\\E_2\{\sigma+\in(\omega\dot{J}-\sigma/2\in)\}&=\frac{\partial H_2}{\partial x_1}-\frac{\partial H_1}{\partial x_2}\end{aligned}\tag{27}$$

and

$$\begin{aligned}\mu\{-\omega\dot{J}+\sigma/2\in\}H_1&=\frac{\partial E_3}{\partial x_2}\\\mu\{-\omega\dot{J}-\sigma/2\in\}H_2&=\frac{\partial E_3}{\partial x_1}\\\mu\{-\omega\dot{J}+\sigma/2\in\}H_3&=\frac{\partial E_2}{\partial x_1}-\frac{\partial E_1}{\partial x_2}\end{aligned}\tag{28}$$

where $(H_p\wedge E_p)=\Phi_p^H(\rho,\phi,0)\wedge\Phi_p^E(\rho,\phi,t)$ are time dependent wave functions satisfying the relations

$$\Phi_p^H(\rho,\phi,t)=\left(\sum_{i=1}^{\infty}\boldsymbol{A}_i{}^p(H)J_\eta(\rho k_i)e^{n\phi J}\right)e^{(j\omega t-\sigma t/2\in)}\tag{29}$$

and $\Phi_p^E(\rho,\phi,t)=\left(\sum_{i=1}^{\infty}\boldsymbol{A}_i{}^p(E)J_\eta(\rho k_i)e^{n\phi j}\right)e^{(j\omega t-\sigma t/2\in)}$

$$E_1=\frac{1}{J\omega\in}\frac{\partial H_3}{\partial x_2}\tag{30}$$

$$E_2=\frac{1}{J\omega\in}\frac{\partial H_3}{\partial x_1}\tag{31}$$

$$E_3 = \frac{1}{J\omega \in}\left(\frac{\partial H_2}{\partial x_1} - \frac{\partial H_1}{\partial x_2}\right) \tag{32}$$

and

$$H_1 = \frac{-1}{\mu\omega J}\frac{\partial E_3}{\partial x_2} \tag{33}$$

$$H_2 = \frac{-1}{\mu\omega J}\frac{\partial E_3}{\partial x_1} \tag{34}$$

$$H_3 = \frac{-1}{\mu\omega J}\left(\frac{\partial E_2}{\partial x_1} - \frac{\partial E_1}{\partial x_2}\right) \tag{35}$$

for perfect dielectric condition $\sigma = 0$

Hence, one can arrive at the following theorems :

Theorem 1 : An axially independent magnetic intensity vector ***H*** is associated with a time dependent cylindrical wave $\Phi_p^H(\rho,\phi,t)$ of frequency 'ω' and the damping factor $(\sigma/2\in)$ iff the bounding surfaces ∂K of the obstacle K is conducting $(\sigma \neq 0)$, and the components of electric intensity vector ***E*** are given by (27) such that the relation $4\in k^2 = \mu\left(4\in^2\omega^2 + \sigma^2\right)$ becomes valid.

Theorem 2: An axially independent electric intensity vector ***E*** is said to be associated with a time dependent cylindrical wave $\Phi^E(\rho,\phi,t)$ of frequency ω and the damping factor $(\sigma/2\in)$ iff the bounding factor ∂K of the obstacle K is conducting $(\sigma = 0)$, and the components of magnetic intensity vector ***H*** are given by (28) such that the relation $4\in k^2 = \mu\left(4\in^2\omega^2 + \sigma^2\right)$ becomes valid.

Determination of current density *J*:

A current density is constituted by the conduction current ***J***$_c$ and the displacement current ***J***$_d$ according to Maxwell's theorem in electromagnetics and thus one can express ***J*** in the form

$$\boldsymbol{J} = \boldsymbol{J}_c + \boldsymbol{J}_d = \sigma\boldsymbol{E}(\rho,\phi,t) + \in\frac{\partial\boldsymbol{E}}{\partial t}(\rho,\phi,t) \tag{36}$$

Now, combining the relation (26) and (36), ***J*** may be finally expressed in the following form

$$\boldsymbol{J} = \phi^{E}(\rho,\phi)\overline{e}^{t((\sigma/2\in)-j\omega)}(\sigma/2 + j\omega\in) \qquad \left(j=\sqrt{-1}\right) \qquad (37)$$

where, $\Phi^{F}(\rho,\phi)$ stands for a cylindrical wave function associated with the electric field intensity ***E*** is given by the relations:

$$\boldsymbol{E}(\rho,\phi,t) = \Phi^{E}(\rho,\phi)\exp\{(j\omega-(\sigma/2\in))\}t$$

and

$$\Phi^{E}(\rho,\phi) = \sum_{i\in J^{+}} \boldsymbol{A}_i(F)J_{\eta}(\rho k_i)e^{\eta\phi j}$$

3. Conclusions

The present paper gives an interaction of an axially independent EM field associated with an echellets model. The model happens to be vital part of a periodic echellete antenna forming a corrugated structure. The present field of study happens to be equivalent to EM boundary value problems. Two important EM problems due to Dirichlet and Neumann have been taken into consideration subject to the prescribed values of the said EM field and its normal derivatives on the boundaries of the model. The wave nature of the present EM field has been justified by arriving at the non-linear relation

$$4\mu\in k^2 = \mu^2\sigma^2 + 4\omega^2\mu^2\in^2$$

The governing Maxwell's equations have been encountered for finding the magnetic field intensity and the electric field intensity vectors subject to Dirichlet's and Neumann's boundary conditions on the outer surfaces of the said model. Finally the results have been used for computing the current density.

References

[1] K N Bhowmick and K K Dey *J. Electronics and Telecom. Engineers* **30** 112 (1984)

[2] K N Bhowmick and Sanjay Kumar *Indian J. Phys.* **65B** (4) 329 (1991)

[3] Sanjay Kumar *Proc. of Math. Soc., B.H.U* **26** 35 (2010)

[4] Sanjay Kumar *Proc. of Math. Soc., B.H.U* **25** 71 (2009)

[5] S K Raghuwanshi *Indian J. Phys.* **84** 831 (2010)

[6] S I Ghobrial and M S Sami *IEEE Trans. Antenna Propagation* **Ap35** 418 (1987)

[7] L O Roderice, V R David and B H Daniel *IEEE Trans. Antenna Propagation* **Ap26** 318 (1978)

[8] H Kreiss *Comm. Pure Appl. Math.* **23** 277 (1970)

[9] R Sakamoto *J. Math. Kyoto Univ.* **10** 349 (1970)

[10] I Hossain et. al. *Indian J. Phys.***88**(1) 5 (2014)

[11] G N Watson *Theory of Bessel Functions 2nd ed* 398 (1958)

[12] Wei K.P. et al *Progress in Electromagnetic Research* **126** 101 (2012)

[13] K N Bhowmick *J. Electronics and Telecom. Engineers* **34** (4) 319 (1988)

[14] I N Sneddon *Special functions of Mathematical Physics and Chemistry, 2nd ed* 133 (1961).

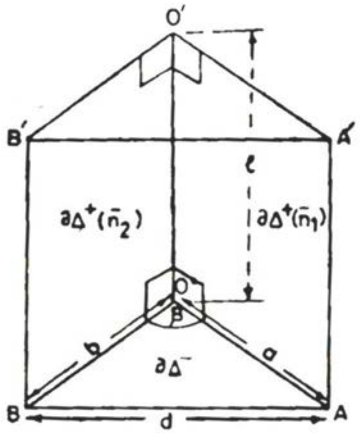

Figure 1

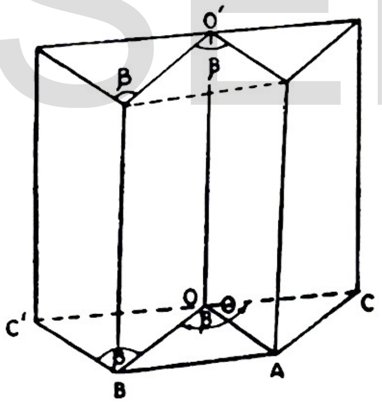

Figure 2

Captions of the Figures

Figure 1.

A convex triangular prism of dimensions a, b, d and with it's flare angle 'β', OO' is perpendicular to the planes Δ^{s} OAB and $O'A'B'$.

Figure 2.

A model 'M' consists of a triangular prism formed by Δ^{s} OAB and $O'A'B'$ and its adjacent groove regions formed by the sides BC' and AC and the sides parallel to OO', OA and OB.